High schoolers excel at Oxford post-graduate quantum exam: experimental evidence in support of quantum picturalism

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This is a ***non-proceedings submission****, associated with our experiment we finalised over the past summer. Available resources include a published paper describing the experiment and additional educational context:*

https://ieeexplore.ieee.org/abstract/document/10313724

Quantinuum blog post giving the results:

https://medium.com/quantinuum/everyone-can-learn-quantum-now-even-at-a-cutting-edge-level-and-we-have-the-test-scores-to-prove-49e7fdc5c509

The Guardian/Oberver coverage of the results:

https://www.theguardian.com/science/2023/dec/16/physicist-bob-coecke-its-easier-to-convince-kids-than-adults-about-quantum-mechanics

We are at the dawn of a second quantum revolution where quantum is not just used in order to produce the bits in our computers, but where quantum systems become the computers themselves. The impact it will have on, for example, chemistry, microbiology, medicine, material science, communications and AI, is likely to change the world as we know it. Governments have already declared quantum to be of geo-political importance second to none, and industries and the financial sector as of vital importance for survival. Unfortunately, quantum is often regarded as only comprehensible for those with an exceptional intellect and who also have enjoyed the privilege of an advanced dedicated education.

The language in which quantum is traditionally formulated, following von Neumann [25], is the language of complex Hilbert spaces, tensor products thereof, and maps between those (HilbS). This subject is usually thought in advanced undergraduate level mathematics and physics courses. Applications that lead to new quantum technologies are mostly thought at post-graduate level. Interestingly, von Neumann himself denounced 'his own language for quantum' merely three years after publishing his book on it [26], and dedicated much time thereafter to finding an alternative language [22].

This article is concerned with a new language for quantum, to which we refer as *quantum picturalism* (QPict) [5]. It is the subject of two books written by some of the authors, respectively entitled Picturing Quantum Processes [10] and Quantum in Pictures (QiP) [9]. The first one is the text book of an Oxford University postgraduate course that has been running for well over ten years now. The second one, remarkably, has no mathematical prerequisites beyond what is already taught to 6-7 year olds in the UK, namely angles [24]. Still, QiP covers advanced quantum topics, some of which that only have been discovered in the past few decades, and some even only in the past few years. In fact, some of the latter are not present in any other quantum textbooks we know of.

Initially, QPict was not intended as an educational tool, but as a high-level quantum language [1] and engineering tool [6], and for this purpose it has been adopted widely within academic and industry quantum ecosytems [14, 13, 18, 20, 17]. In this abstract we provide compelling evidence that QPict enables one to teach advanced quantum concepts at high school level. The work has already had substantial impact, with a number of governments now seriously considering to use quantum picturalism in order to start teaching quantum at secondary school level.

We therefore now have a tool to make quantum education more inclusive than ever imagined, and in a manner that goes hand-in-hand with cutting-edge technology development. The impact that this will have cannot be underestimated. As we are at the start of an important new technological revolution, it is of vital importance to develop a newly trained work force, for stakeholders in business and governance to make the right responsible decisions, and to make society more broadly quantum-ready, across different age groups, educational backgrounds, and social backgrounds. We can contrast this with the current situation in AI technology, where society nor the stakeholders understand what is going on, as even the specialists and business leaders involved cannot even explain the workings of their systems.

Course language. While QiP only requires mathematics that is already thought to 6-7 year olds in the UK, it is not mathematics-free, but employs a new very intuitive kind of mathematics. Historically, this new kind of mathematics emerged from a field called category theory, and a particular branch thereof called monoidal categories [2, 16, 3, 19]. In [15] it was shown that in their most basic form these are equivalent to a pictorial notation introduced by Penrose [21]. QPict shows that this pictorial notation can be extended capturing all of the essential quantum notions like entanglement, mixed states and processes, and observables and their complementarity [4, 23, 11, 12, 7, 8].

Course materials. The course book for the course was QiP, and it covers the following concepts, stated here in HilbS terms (IOOA): tensor & Kronecker product, Bell states/effects, transpose, inner-product, unitarity, observables (in particular Z and X), phases, complementarity, bits, Hadamard gate, phase gates, CNOT-gate, non-determinism, trace, measurement, encoding classical data, mixed states, CPTP maps, decoherence, classical correlations, uncertainty, strong complementarity (non-standard in HilbS), probabilities. It covers the following subjects, all in standard terminology (IOOA): quantum teleportation, entanglement swapping, quantum circuits, circuit optimisation, MBQC, causality (non-standard in HilbS), no-signalling-from-the-future, no-signalling, QKD, measurement disturbance, no broadcasting, phase gadgets for circuit optimisation, non-locality in GHZ form.

In addition to the textbook, we also filmed blackboard lectures covering all the relevant material. Additional materials also included exercises with filmed solutions. All of these soon will be made publicly available, after post-processing.

Course structure. The structure of the course mimicked that of an Oxford University postgraduate course as close as possible. Spanning eight weeks, students were asked to watch one video lecture each week. In an Oxford graduate course there would be two to three in-person lectures each week. In the same week there was a 1-hour session with a tutor, in groups typically of 10 students, that enabled Q&A as well as some guide exercises, just like in Oxford graduate courses. Additional 'take home' exercises with the filmed solutions were also released during these 8 weeks, 12 such exercises in total, and the students were given about a week to try to do them, before solutions were released. In Oxford University postgraduate courses there also are take home exercises.

There were some clear disadvantages for the takers of the course as compared to those of an Oxford University postgraduate course: (1) Everything was online. (2) It was a 1st of its kind so a learning process for us as well. (3) Since we were working with minors, ethics restrictions impeded many features of a typical teaching experience, like student interaction. We were moreover not allowed to use any social platforms, which is now common place for students to interact. (4) The students didn't have any direct consequences if they performed badly, as we gave a certificate to everyone that took the course. (5) The period of teaching included A-level exams, regular school, and family summer holidays. (6) The tutors found it particularly hard to teach this generation, who grew up with smartphones in a Covid-era. Hardly any student used camera, and questions were typed.

Student recruiting. The students were recruited through a flyer, as on the last page here, that was distributed on social media and through schools. The flyer stated the promise that everyone that takes the course gets a certificate and a signed limited edition if QiP, independent of the results obtained. We received almost 1000 applications, and after an initial screening against the selection criteria (being a high school student and in the right age group) we randomly selected 60 students.

The exam. The structure of the exam also mimicked that of an Oxford University postgraduate exam. The exam was triple marked and if marks deviated too much, a fourth marker was involved. Such an exam takes the form of a take home exam for which two to three weeks time for solving it is awarded. All of our questions were taken from Oxford postgraduate exams, and there were three questions each with several parts, which typically are of increasing difficulty.

The first question concerned quantum circuit simplification. Question 1a required the application of a four-qubit circuit to a state. It can be solved fairly straightforwardly both in QPict (using fusion and copy) and in HilbS (applying gates one-by-one). Question 1b required showing that a three-qubit circuit reduces to the identity. In QPict it requires some more rules (leg-chop and colour-change), and in HilbS multiplying 8x8 matrices for verification which becomes tedious. When increasing the number of qubits the complexity for QPict would stay the same while for HilbS it would become exponentially harder. Question 1c asks what a three-qubit circuit simplifies to. In QPict it requires one more rule (square-pop/bialgebra), but in HilbS this question would be essentially impossible to solve as matrices don't enable rewriting.

The second question concerned multi-party protocols. Questions 2a and 2b are much more intuitive to solve with QPict than in HilbS. Question 2c is very hard in HilbS as it requires an impossibility argument.

The third question is about computing correlations when measuring quantum states. Questions 3a.1 and 3a.2 are fairly easy in HilbS, while in QPict they are 'canary questions': if one can't solve these ones then hasn't made any effort at all. Concretely, the question simply asked for two spiders to either be fused, and to be leg-chopped (cf. Hopf law) respectively. Questions 3a.3 and 3a.4 require mixed states in HilbS, while in QPict they require using the representation of mixed states in terms of 'doubling'. Question 3b is difficult even in QPict, so gives us an indication of which students are particularly smart, and is sometimes called a 'sentinel question'.

Results. These are the results for the 51 students that completed the course:

- 80% students passed
- 48% students had a distinction
- 11 students failed the canary questions
- 12 students passed the sentinel question

These results are better than typical Oxford University exam averages.

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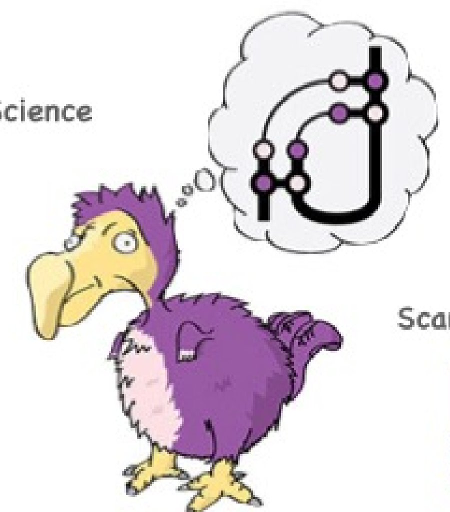

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